A synthesis of technical analysis and fractal geometry: Evidence from the Dow Jones Industrial Average components

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Abstract

The profitability of technical analysis has been investigated extensively, with inconsistent results. This paper seeks to develop new insights into the profitability of technical trading rules through a synthesis of fractal geometry and technical analysis. The Hurst exponent (H) emerged from fractal geometry as a means to detect long-term dependencies in a time series; the same dependencies that technical analysis should be able to identify and exploit to earn profits. Two tests of the synthesis are conducted using the thirty Dow Jones Industrial Average components. Firstly, the financial series are classified into three groups based on their H to determine if a higher (lower) H results in higher returns to trending (contrarian) trading rules. Secondly, the relationship between H and profits to technical analysis are estimated through OLS regression. Both tests suggest that the fractal nature of a time series explains a significant portion of the profits generated by technical analysis.

Keywords: Technical analysis; Rescaled range analysis; Hurst exponent; Long-term dependencies; Market efficiency.
JEL Classification: C4; C22; G14

1. Introduction

The fractal nature of financial data has been investigated throughout economic literature. Fractal geometry provides a technique to identify a time series' long-term dependencies; dependencies that technical analysis should be able to exploit to earn profits. Technical analysis is one of the earliest forms of investment analysis, mainly because stock prices were among the first types of publicly available information. Technical analysis uses past prices to identify patterns that predict future prices. Technical analysis is popular among academics and traders; however, the extant body of literature is inconsistent as many studies signify the informational content of technical analysis (Brock, Lakonishok and LeBaron (1992), Gençay (1999), Lento and Gradojevic (2007), and Lento,

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Gradojevic and Wright (2007)), while other studies support the opposite (Allen and Karjalainen (1999), Lo, Mamaysky and Wang (2000), and Bokhardi et. al. (2005)).

Recently, Hurst's exponent (H) (Hurst, 1951) has emerged from fractal geometry into economics research as a means of classifying a time series based on its long-term dependencies (Peters, 1991 and Peters, 1994). A H of 0.50 indicates a series exhibits Brownian motion. A $0<H<0.5$ indicates an anti-persistent series, suggesting the data set exhibits mean reverting tendencies. A $0.5<H<1$ indicates a persistent series, suggesting the data is trend reinforcing. The strength of the trend increases as H approaches 1. The H thus provides a method of classifying time series, which can be beneficial in identifying which markets have greater predictability.

The purpose of this paper is to develop new insights through a synthesis of technical analysis and fractal geometry to help explain the inconsistent empirical results in the extant body of literature. The synthesis posits the following: fractal geometry provides a technique (H) that detects the long-term dependencies (reinforcing or revering trends) in the historical price data of a time series; the same trends that technical analysis purports to identify and utilize to predict future price movements. Therefore, trending trading rules should be more profitable in markets that exhibit trend reinforcing characteristics, while contrarian trading rules should be more profitable in markets that exhibit anti-persistent, or mean reverting, tendencies. Two empirical tests are conducted to evaluate the synthesis and the resulting relationship between the H and profits to technical analysis. Firstly, the financial series are classified into three groups based on their H ($H<0.5$; $0.5<H>0.55$; $H>0.55$) to determine if time series with a higher (lower) H results in higher returns to trending (contrarian) trading rules. Secondly, OLS regression is used to estimate the relationship between the H and profits to technical analysis. The thirty Dow Jones Industrial Average components provide the sample data.

The results suggest that the H is able to identify long-term dependencies in a time series and these time series result in higher profits to technical analysis. The classification analysis reveals that profits from trending trading rules are higher (average of 11%) for time series that exhibit long-term dependencies (high H) and lower (average negative return of 16.8%) for time series that exhibit anti-persistent trends. The regression analysis results in a significant R^2 of 0.31, revealing that the fractal nature of a time series explains a significant portion of the returns to technical analysis. The moving averages and trading range break-out rules were the best at exploiting the dependencies (R^2 of 0.37 and 0.372 respectively) while the filter rule was inconsistent. The results are consistent with the

synthesis' theory. Sub-period analysis confirms the robustness of the results. The results from the contrarian trading rules are similar to the trending trading rules.

This paper offers a significant contribution by expanding the current literature. This study develops new insights and theories of technical analysis through a synthesis with concepts from fractal geometry. The vast majority of the literature investigating the fractal nature of financial data seeks to determine market predictability (Peters, 1991; Qian and Rasheed, 2004; and Corazza and Mlliaris, 2002); however, the literature does not utilize technical analysis to build on the identification of market predictability to determine if abnormal returns can be generated after accounting for transaction costs. This paper extends the literature by developing a synthesis that seeks to determine if technical trading rules are more profitable in markets that exhibit long-term dependencies. The synthesis provides new insight into the inconsistent empirical results on technical analysis. Furthermore, this study provides an additional examination of moving average, filter, Bollinger Band, and trading ranges break-out rules on the DJIA components that is unique as it assesses the trading rule profits as calculated at various scales.

The remainder of the paper is organized as follows: Section 2 provides a review of the literature, and develops the synthesis and hypotheses; Section 3 describes the data; Section 4 discusses the methodology; Section 5 presents the results; and Section 6 offers a summary and concluding thoughts.

2. Theory and Hypothesis Development

Fractal geometry has been making inroads into economic theory due to the pioneering work of researchers such as Beniot Mandelbrot. Fractal geometry questions, and provides alternatives to, many fundamental assumptions in economic and finance theory and therefore can permeate through many areas of economics, including technical analysis and technical trading rules. Currently, the extant body of research on technical analysis is inconsistent as many studies support its profitability (Brock, Lakonishok, and LeBaron, 1992), while others suggest the opposite (Lo, Mamaysky and Wang, 2000).

A synthesis of technical analysis and fractal geometry can provide fertile grounds for new theory development and insights, along with novel empirical tests, to enhance our understanding of the profitability technical trading rules. The following literature review develops this synthesis and

proceeds as follows: Section 2.1 provides a brief discussion of the technical analysis literature; Section 2.2 provides a discussion of fractal geometry, including its history, the time series dynamic process, and its application to capital markets; and Section 2.3 develops the synthesis and hypotheses.

2.1 – Technical trading rules

Since its inception, research on technical analysis has generally been inconsistent. The first studies were conducted by Alexander (1961 and 1964) and Fama and Blume (1966) who suggest that excess returns cannot be realized by making investment decisions based on filter rules. However, Sweeney (1988) later re-examined the data used by Fama and Blume (1966) and found that filter rules applied to fifteen of the thirty Dow Jones stocks earned excess returns over buy-and-hold alternatives. The excess returns existed for a number of years following the end of the Fama and Blume sample.

Technical trading rules have also been extensively tested in the foreign exchange market, beginning with Dooley and Shafer (1983) who tested filter rules for nine currencies. After accounting for the bid and ask spread, their results suggest that smaller filters were profitable during the period studied for all currencies, while larger filters were also profitable in over half of the sample periods. Sweeney (1986) conducted a similar test that yielded similar results. Schulmeister (1988) analyzed the US/DM spot rate over the periods of 1973 – 1986 by using not only filter rules, but also moving average models and momentum models. Even after adjusting for interest expense and transaction costs, the result indicate that trading rules would have generated profits over the period studied.

The number of studies on technical trading rules significantly increased during the 1990s, along with the methods used to test trading rules. However, the extant body of research was unable to conclude on the profitability of technical analysis. Some of the most influential studies that provide indirect support for trading rules includes Jegadeesh and Titman (1993), Blume, Easley, and O'Hara (1994), Chan, Jagadeesh, and Lakonishok (1996), Lo and MacKinlay (1997), Grundy and Martin (1998), and Rouwenhorst (1998). Stronger evidence can be found in the research of Neftci (1991), Neely, Weller, and Dittman (1997), Chang and Osler (1994), Osler and Chang (1995), Lo and MacKinlay (1997), and Neely and Weller (1998). One of the most influential studies on technical analysis was prepared by Brock, Lakonishok, and LeBaron (1992) who used bootstrapping techniques and two simple, yet popular, trading rules to reveal strong evidence in support of technical analysis's predictive nature.

The results suggest that patterns uncovered by technical rules cannot be explained by first order correlations or by the potential for changing expected returns caused by changes in volatility.

However, not all studies support the efficacy of technical analysis. For example, Allen and Karjalainen (1999) used genetic programming to develop optimal ex ante trading rules for the S&P 500 index. They found no evidence that the rules were able to earn economically significant excess returns over a buy-and-hold strategy during the period of 1970 – 1989. Furthermore, Lo, Mamaysky and Wang (2000), using a smoothing techniques based on nonparametric kernel regression, found that certain technical patterns can provide information when applied to a large number stocks ranging over a vast number of time periods. However, the results do not imply that technical analysis can be used to generate excess trading profits. Rather, the results indicate the possibility that technical trading rules can add value to the investment process and compliment fundamental analysis.

Bokhardi et. al. (2005) investigated the effectiveness of simple trading rules in relation to the size of the firm. Bokhardi segregated a number of companies based on their size and applied trading rules on their past prices for a sample period of 1987 – 2002. The results indicate that trading rules are more effective at predicting the future price movements of firms with smaller capitalization. However, Bokhardi concluded that trading rules cannot be used profitably after adjusting for transaction costs.

2.2 – Fractal Geometry

2.2.1 – Historical Perspective

Fractal geometry has recently emerged into the world of Mathematics as a compliment to Euclidean geometry as an attempt to better explain and describe the objects and shapes of the real world. The term fractal was coined by Benoit Mandelbrot, who is largely responsible for the present day interest in fractal geometry and chaos theory. Chaos theory attempts to use nonlinear dynamic systems to provide order to what is perceived to be random.

During the 1960's Benoit Mandelbrot believed that securities returns followed a fractal time series and that Brownian motion was not an adequate statistical description of the true stochastic process generating securities returns. In order to resolve this inadequacy, Mandelbrot worked in two perpendicular directions to expand the class of fractal time series. One direction involved relaxing an assumption of finite variance, which introduces what Mandelbrot termed the "Noah Effect." The other direction entailed relaxing an independence assumption, thereby allowing for a "Joseph

Effect." (Mandelbrot, 1972). The Noah Effect (recalling the Biblical account of the great deluge) refers to the tendency of various time series with presumably independent increments, especially speculative time series, to exhibit abrupt and discontinuous changes. The Joseph Effect is named after the biblical story in which Joseph prophesied that the residents of Egypt would face seven years of feast followed by seven year of famine[2].

The Joseph effect denotes the property of certain time series to exhibit persistent behavior (such as years of flooding followed by years of drought along the Nile River basin) more frequently than would be expected if the series were completely random, but without exhibiting any significant short-term (Markovian) dependence. To describe such processes, Mandelbrot broadened the idea of Brownian motion into the class of stochastic processes called "fractional" Brownian motion (fBm).

2.2.2 – Chaotic dynamical process

A white noise process is the statistical paradigm against which the sequence of increments from a chaotic dynamical process is typically contrasted. White noise traditionally refers to a sequence whose increments are independently and identically distributed with zero mean and finite variance. Brownian motion is a well-known paradigm in finance that can be described as a white noise process for which the independent increments are identically normally distributed. Brownian motion underlies most of modern finance theory's most important contributions.

A fractal time series is statistically self-similar regardless of the time frame over which the increments of the series are observed, aside from its scale. For example, a time series of daily, weekly, monthly, or yearly observations would exhibit similar statistical characteristics. Schroeder (1991) notes that the paradigm of random fractals can be described as Brownian motion, as a white noise process, that exhibits these scaling time series properties.

Fractional Brownian motions exhibit complicated long-term dependencies that can be characterized by the Hurst exponent (H). The H denotes the level of long-range dependence in data and generally ranges from 0 to 1 (Hurst, 1965). Additionally, the power spectrum of the increments of fractional Brownian motion is proportional to f^{β}, where $\beta = 2H-1$, so that Brownian motion as a white noise paradigm has a flat spectrum (Feder, 1988).

[2] Mandelbrot (2004) provides a detailed history and discussion of the Hurst exponent, the Joseph effect, and Noah effect.

If $0.5<H<1$, then the series will exhibit persistence, with fewer reversals and longer trends than the increments of Brownian motion. In this case, the graph would appear smoother than that of a random walk. Figure 1 – Panel A presents a graph of McDonald's weekly price time series which has a H of 0.629. In addition, the power spectrum for such a series would be proportional to f^{β}, where $\beta>0$, so that the series would be subject to long-range dependence. On the other hand, if $0<H<0.5$, then the series will exhibit anti-persistence, as evidenced by a greater number of reversals and fewer and shorter trends than in a white noise series. Visually, a graph of such a series would appear more jagged than a random walk. Figure 1 – Panel B presents a graph of J&J's weekly price time series which has a H of 0.464. Figure 1 – Panel C presents a graph of Alcoa's weekly price time series which has a H of 0.498, closely resembling what would be expected from Brownian motion.

Insert Figure 1 about here

In order to measure the dependencies that a time series most closely resembles, Mandelbrot developed a statistical technique called rescaled range analysis that yields a measure of Hurst's exponent. This involves comparing a linear measure of the spread of the time series (a variation of its sample range) to a quadratic measure (its sample standard deviation). Using the rescaled range analysis, Greene and Fielitz (1977) found considerable evidence of temporal dependence in daily stock returns for the period December 23, 1963 to November 29, 1968, after accounting for short-term linear dependencies (autocorrelation) within the data.

2.2.2 – Capital market research

Fractal geometry has not been researched in the capital markets as extensively as the theories of modern finance. There are no known studies that investigate the relationship between technical analysis and fractal geometry. The vast majority of the literature investigates the Hurst exponent's ability to identify financial market predictability; however, the literature does not use technical analysis to build on the identification of financial market predictability to determine if abnormal returns can be generated after accounting for transaction costs.

The most popular example is by Peters (1991) who estimates the Hurst exponent to be 0.778 for monthly returns on the S&P 500 from January 1950 to July 1988. For a sample of individual stocks,

Peters found Hurst exponents ranging from 0.75 for Apple Computer down to 0.54 for Consolidated Edison. All of these values are greater than 0.5, indicating a greater persistence among stock returns than would be expected if stock prices followed a geometric Brownian motion process.

The *H* was investigated in the foreign exchange market for the British Pound, the Canadian Dollar, the German Mark, the Swiss Franc, and the Japanese Yen by Corazza and Malliaris (2002). They found that in the majority of the samples studied, the foreign currency markets exhibit a *H* that is statistically different from 0.5. Furthermore, they also found that the *H* is not fixed but it changes dynamically over time. The interpretation of these results is that the foreign currency returns follow either a fractional Brownian motion or a Pareto- Levy stable distribution.

Qian and Rasheed (2004) classified various series of financial data representing different periods of time and experimented with backpropagation neural networks to show that series with large *H* can be predicted more accurately than those with *H* close to 0.50. The authors concluded that the *H* provides a measure for predictability.

More recently, Hodges (2006) and Bender et al. (2006) began investigating the possibility of developing portfolios based on identifiable long-term dependencies. Hodges (2006), for example, examines an investor's ability to form arbitrage portfolios under realistic transactions costs for values of H very different from .5. Bender et al. also seek to develop a general theory of arbitrage portfolio building based on a long term dependent processes. However, both these papers do not specifically focus on investigating the efficacy of technical analysis in light of long-term dependencies.

Glenn (2007) also investigated the Hurst exponent and long term dependency on the NASDAQ. Using the rescaled range analysis, a *H* of 0.59 was calculated for 1-day returns on the NASDAQ. It is interesting to note that the *H* increased monotonically to a value of 0.87 for 250-day (annual) returns. Most of this increase was also observed in simulated returns derived via a Gaussian random walk.

There are various scholars who rebut the *H* ability to identify long term dependencies by arguing that the rescaled range analysis is skewed. Specifically, issues with the sensitivity of the *H* to short term memory, the effects of the pre-asymptotic behaviour on the significance of the *H* estimate and the problems with structural changes (see, Lo (1991), Ambrose et al. (1993), or Chueng (1993)) have be raised. The most significant rebuttal was offered by Lo (1991), who published a paper refuting

Mandelbrot's claims for *H*. Lo reported that the rescaled range analysis could confuse long-term memory with the effects of short term memory. Along these lines, Jacobsen (1996) investigated the return series of five European countries, the United States and Japan using the modified rescaled range statistics, as introduced by Lo (1991) and concludes that no long-term dependence exists.

However, since Lo's publication, many economists have reported that his tests were potentially flawed (Mandelbrot, 2004). Additionally, a number of new contributions suggested alternative techniques for the estimation of a pathwise version of *H* eliminate the lack of reliability in the *H* (Bianci, 2005 and Carbone et al., 2004).

2.3 –A synthesis of technical analysis and fractal geometry

The extant body of literature provides inconclusive evidence on the profitability of technical trading rules. Furthermore, the literature on the fractal nature of financial markets appears to stop once dependencies have been identified or refuted. New insights into technical analysis can be obtained by extending the literature through a synthesis of fractal geometry and technical analysis.

Technical trading rules are based on the premise that time series exhibit certain patterns in its past data that can be used to predict future movements. It can therefore be deducted that trending technical trading rules (e.g., filter rule, trading-range break-out, and moving average rules) should be more profitable on time series that exhibit long-term persistence or dependencies, as postulated by Mandelbrot with the level of the Nile River. Conversely, time series that are anti-persistent should not provide fruitful results to trending technical trading rules as there are no continuing patterns in the time series to identify and exploit. The *H* can be used to identify the dynamic processes of a time series. Therefore, based on this synthesis, identifying the dependencies in a time series motion should be able to partially explain the inconsistent and conflicting results evident in the extant body of literature. The synthesis of fractal geometry and technical analysis provides the first hypothesis:

H_1: The profitability of trending technical trading rules should be higher on time series that have higher Hurst exponents and lower on time series that have lower Hurst exponents.

As opposed to trending technical trading rules, investors may employ a contrarian trading rule. A contrarian rule attempts to exploit a reversal pattern in a time series and essentially sells into

strength (expectation of price decline after an increase) and buys into weakness (expectation of a price increase after a decrease). It can therefore be deducted that contrarian technical trading rules (e.g., Bollinger Bands) should be more profitable on time series that exhibit anti-persistence. Conversely, time series that are persistent should not provide as profitable results to contrarian technical trading rules as there are no continuing reversal patterns in the time series to identify and exploit. This reasoning leads to the second hypothesis:

H_2: The profitability of contrarian trading rules should be higher on time series that have lower Hurst exponents and higher on time series that have lower Hurst exponents.

Hypotheses one and two investigate the relationship between the *H* and the profits from trending and contrarian technical trading rules; however, accepting the hypotheses does not suggest that investors can successfully utilize technical analysis to earn abnormal profits by understanding a time series fractal nature because both the *H* and the profits will be calculated on the same dataset. Therefore, an additional hypothesis, with a different test, is required to understand whether traders can successfully employ a trading strategy that uses an observed *H* to correctly employ a trading rule.

H_3: The lagged H (H_{t-1}) can predict whether a contrarian or trending trading rule will be profitable for a time-series.

Testing the first three hypotheses will make a significant contribute as there are no other known studies that test the relationship between profits from technical analysis and long-term dependencies. Aside from the main hypotheses, an ancillary hypothesis will be tested with the intent of better understanding the fractal nature of technical analysis. A fourth hypothesis will be tested regarding the scale of the time series used to generate the trading signals.

There is very little empirical evidence on the effectiveness of the trading rules at various scales (e.g. daily, weekly, monthly). Brownian motion suggests that independent increments are identically normally distributed, whereas pure fractal Brownian motion suggests that a time series is statistically "self-similar" (apart from scale) regardless of the time frame over which the increments of the series are observed. There is a vast amount of literature that discusses the non-normality and lack of Brownian motion of stock returns (Cootner 1964, Fama 1965, Officer 1972). If the stock returns exhibit the characteristics of a fractal time series, rather than Browian motion, there should be no

difference in the effectiveness of technical trading rules with data at different scales. This leads to the development of the third hypothesis:

H_4: There is no difference between the profitability of technical trading rules on the same data set when calculated with different time scales.

3. The data

The *H* and the trading rule profits are calculated on all thirty stocks that compose the DJIA (as at July 2008) for the ten year period of July 1998 to July 2008. Trading rules can be calculated at various data frequencies. The data frequency selected depends on different factors and preferences. Investors can use high-frequency data or longer horizons. This study utilizes daily and weekly data. Daily data will be used because a typical off floor trader will most likely use daily data (Kaastra and Boyd, 1996). Furthermore, intraday time series can be extremely noisy. Along these lines, weekly data will also be used as it is readily available to all traders. The number of daily and weekly observations in each data set provides a sufficient number of observations to allow for the formation, recurrence and investigation of the trade rule signals and for the estimation of the *H*.

The use of raw daily price data in the stock market has many problems as movements are generally non-stationary (Mehta, 1995), which interferes with the estimation of the *H*. The market index series are transformed into rates of return to overcome these problems. Given the price level P_1, P_2, … P_t, the rate of return at time t is transformed by:

(1) $$r_t = \log(p_t) - \log(p_{t-1})$$

where p_t denotes the spot price (stock market indices or the exchange rate). The descriptive statistics for the thirty DJIA components are presented in Table 1.

Insert TABLE 1 Here

4. Methodology

The individual and average profits from twelve trading rules, along with the *H* are calculated for all thirty stocks. Profitability is defined as the returns from the trading rules less the buy-and-hold strategy returns, adjusted for transaction costs. Therefore, by definition, profits can also be negative.

4.1 Trading rules

The trending trading rules are the moving-average cross-over rule (MACO), filter rule, and trading range break-out rule (TRBO), while the contrarian trading rule will be the Bollinger Band.

A MACO rule tries to identify a trend by comparing a short moving average to a long moving average. The MACO generates a buy (sell) signal whenever the short moving average is above (below) the long moving average. This study tests the MACO rule based on the following signals:

$$(2) \quad \frac{\sum_{s=1}^{S} R_{i,t}}{S} > \frac{\sum_{l=1}^{L} R_{i,t-1}}{L} = Buy$$

$$(3) \quad \frac{\sum_{s=1}^{S} R_{i,t}}{S} < \frac{\sum_{l=1}^{L} R_{i,t-1}}{L} = Sell$$

where $R_{i,t}$ is the log return given the short period of S, and $R_{i,t-1}$ is the log return over the long period L. The following short, long combinations will be tested: (1, 50), (1, 200) and (5, 150).

Filter rules generate signals based on the following logic: Buy when the price rises by *f* percent above the most recent trough and sell when the price falls *f* percent below its most recent peak. This study tests the filter rule based on three parameters: 1%, 2%, and 5%.

The TRBO generates a buy signal when the price breaks-out above the resistance level and a sell signal when the price breaks below the support level. The resistance/support level is defined as the local maximum/minimum. The TRBO rule is examined by calculating the local maximum and minimum based on 50, 150 and 200 days as defined as follows:

$$(4) \quad \begin{aligned} Pos_{t+1} &= Buy, \quad \text{if } P_t > Max\ \{P_{t-1}, P_{t-2}, \ldots, P_{t-n}\} \\ Pos_{t+1} &= Pos_t, \quad \text{if } P_t > Min\ \{P_{t-1}, P_{t-2}, \ldots, P_{t-n}\} \leq P_t \leq Max\ \{P_{t-1}, P_{t-2}, \ldots, P_{t-n}\} \\ Pos_{t+1} &= Sell, \quad \text{if } P_t < Min\ \{P_{t-1}, P_{t-2}, \ldots, P_{t-n}\} \end{aligned}$$

where P_t is the stock price at time t.

Creating Bollinger Bands (BB) requires two parameters: the 20-day moving average (MA20) and the standard deviation (σ) of the 20-day moving average line (σ MA20). The BB is a contrarian trading rule because a sell signal is generated if the price of the security exceeds the 20-day moving average plus two standard deviations (i.e. the market is said to be overextended). A buy signal is generated if the price of the security is less than the 20-day moving average minus two standard deviations. In this case, the market is said to be oversold. BB are traditionally calculated based on a 20-day moving average, +/- 2σ; denoted by BB(20,2). This traditional definition is tested along with two variants: 30-day moving average, +/- 2σ and 20-day moving average, +/- 1σ. A 30-days average is used to determine whether a longer time frame can generate more informative signals. Conversely, +/- 1σ, as opposed to 2σ, is used to determine whether a narrower band can generate more precise signals.

Statistical significance of the trading rules is determined through a bootstrapping methodology as developed by Levich and Thomas (1993). The bootstrap approach does not make any assumptions regarding the distribution of the generating function. Rather, the distribution of the generating function is determined empirically through numerical simulations. The data sets of raw closing prices, with the length $N + 1$, correspond to a set of log price changes of length N. $M = N!$ separate sequences can be arranged from the log price changes with a length of N. Each of the sequence ($m = 1, \ldots, M$) will corresponding to a unique profit measure ($X\,[m, r]$) for each variant trading rule (r for $r = 1, \ldots, R$.) used in this study. Therefore a new series can be generated by randomly rearranging the log price changes of the original data set.

By utilizing the sequence of price changes, the starting and ending price points of the randomly generated time series are forced to be exactly the same as the their values in the original data set. Furthermore, by rearranging the original log price changes, the randomly generated data sets are forced to maintain the identical distributional properties as the original data set. However, the time series properties are random. This simulation can generate one of the various notional paths that the security could have taken from time t (original level) until time t + n (ending day), while maintaining the original distribution of price changes.

The simulation process of randomly mixing the log price returns is repeated 10,000 times for each data set, resulting in 10,000 identically and independently distributed (i.i.d.) representations from the m = 1 , … , M possible sequences. All of the randomly generated data sets have the identical

distributional properties as the original data set; however, the time series properties are random for each data set and are independently drawn from any other notional path.

Each technical trading rule (MACO, filter rule, and TRBO) is then applied to each of the 10,000 random series and the profits X[m, r] are measured. This process generates an empirical distribution of the profits. The profits calculated on the original data set are then compared to the profits from the randomly generated data sets.

The null hypothesis states that if the trading rules provide no useful information, then the profits resulting from trading in the original data sets should not be significantly different from the profits resulting from the randomly generated data sets. If the profits resulting from the original data set are greater than α percent threshold level of the empirical distribution, then the null hypothesis will be rejected at the α percent level (Levich and Thomas 1993).

4.2 The Hurst Exponent (*H*)

The basis of the rescaled range analysis was laid by Hurst et al. (1965). Mandelbrot and Wallis (1968, 1969a, 1969b, and 1971) examined and further elaborated the method. The following is a brief discussion of the rescaled range analysis and the Hurst exponent calculation. The stochastic process of fractional Brownian motion (fBm) occurs when the second order moments of the increments scale as follows:

(5) $$E = \{ (X(t_2) - X(t_1))^2 \} \propto |t_2 - t_1|^{2H}$$

with $H \ni [0, 1]$. The Brownian motion is then the particular case where $H = 0.5$. The exponent H is called the Hurst exponent.

The H measures dependencies in time series non-stochastic motion and is calculated through rescaled range analysis (R/S analysis). For a time series where $X = X_1, X_2, \ldots, X_n$, R/S analysis can be calculated by firstly determining the mean value m, followed by the mean adjusted series Y:

(6) $$m = \frac{1}{n}\sum_{i=1}^{n} X_i$$

(7) $$Y_t = X_t - m, \qquad t = 1, 2, \ldots, n$$

Thirdly, the cumulative deviate series Z is calculated:

(8) $$Z_t = \sum_{i=1}^{t} Y_i \,, \qquad t = 1, 2, \ldots, n$$

Fourthly, the range series R is calculated:

(9) $$R_t = max(Z_1, Z_2, \ldots, Z_t) - min(Z_1, Z_2, \ldots, Z_t) \qquad t = 1, 2, \ldots, n$$

Fifthly, the standard deviation series S is calculated:

(10) $$S_t = \sqrt{\frac{1}{t}\sum_{i=1}^{t}(X_i - u)^2} \qquad t = 1, 2, \ldots, n$$

where u is the mean from X_1 to X_t

Finally, the rescaled range series (R/S) can be estimated:

(11) $$(R/S)_t = R_t / S_t \qquad t = 1, 2, \ldots, n$$

Figure 1 graphically presents the rescaled range analysis that is used to estimate the Hurst exponent on the Dow Jones Industrial Average time series.

Insert Figure 2 about here

4.3 – Testing the synthesis

Two empirical tests are conducted to evaluate the synthesis. A classification test will group each DJIA component based on its estimated *H*. Three groups will be developed as follows: (1) $0.5 > H$; (2) $0.5 < H > 0.55$; and (3) $0.55 < H$. The synthesis postulates that technical analysis should generate higher profits for group three, and lower profits for group one. A quasi- contingency table will present the average profits for each group.

The second test uses OLS regression to estimate the statistical relation between profits from technical analysis and the *H*. To test whether data sets that exhibit higher *H* result in higher profits from technical analysis, the following equation is estimated:

(12) $\text{Profits}_i = a_o + a_1 H_i + \varepsilon_1$

where Profits_i represents the returns in excess of the buy-and-hold trading strategy for DJIA component i, and H_i represents the Hurst exponent for DJIA component i. The intercept is expected to be positive for Hypothesis 1 and negative for Hypothesis 2.

5. Results

5.1 – Profits from technical analysis on the DJIA components

The profits from the technical trading rules and the *H* for each DJIA component are presented in Table 2 with daily data (Panel A – trending rules and Panel B – contrarian rules) and Table 3 with weekly data (Panel A – trending rules and Panel B – contrarian rules). Calculated with daily data, the *H* ranges from a low of 0.452 (Pfizer) to a high 0.587 (Citigroup). Weekly data results in a wider range of *H* (0.431 to 0.629) as Pfizer remains the data sets with the least dependencies, while McDonalds' stock exhibits the most persistency.

Insert Table 2 and 3 about here

On average, the trending trading rules were profitable on seven of the thirty components when calculated with daily data (AIG, C, GM, HD, HPQ, INTC, and MRK). The trending trading rules were the most profitable for GM's stock, as all nine variants calculated with daily data were able to beat a buy-and-hold strategy. The average profit from all trending trading rules is 15.91% for GM's stock. AIG and Intel's stocks were the second and third most profitable time series with average profits of 8.20% and 4.30% respectively generated by the trending trading rules. Trending trading rules were the least profitable for the Exxon Mobile, earning negative 16.84%, followed by the stocks of Wal-Mart and Chevron Corporation (average negative profit of 15.12% and 14.48% respectively).

Calculating the trending trading rules with weekly data resulted in average profits on thirteen of the thirty components (AIG, BAC, C, GE, GM, HD, HPQ, INTC, MCD, MRK, MSFT, PFE, and T).

Again, the trending trading rules were the most profitable for General Motors stock when calculated with weekly data, as all nine variants tested were able to out-perform the buy-and-hold trading strategy. The average profit from the trending trading rules is 41.9% for GMs' stock. Citigroup and AIG's stocks were the second and third most profitable time series with average profits of 28.4% and 27.3% respectively generated by the trading rules. Conversely, weekly data tests were the least profitable for the Exxon Mobile stock, earning negative returns of 52%. Exxon Mobile was also the least profitable with daily data. Caterpillar and Wal-Mart resulted in the second and third least profitable generation (49.7% and 41.5% respectively).

The trending technical trading rules were profitable on seven DJIA components (AIG, C, GM, HD, HPQ, INTL, and MRK) with both daily and weekly data. Therefore, all seven stocks that were profitable with daily data were also profitable when calculated with weekly data. Profits were generated on five additional stocks (BAC, GE, MCD, MSFT, and T) with weekly data.

It is interesting to note that the MAC-O and TRB-O were much more effective than the filter rules. The filter rules generated an average negative profit of 10.59% (daily) and 22.6% (weekly). The filter rules were highly profitable for GM, with the 5% filter earning profits in excess of 78.3%; however, the filter rules performed very poorly on most other DJIA components. The filter rules poor performance is consistent with prior studies (Szakmary, Davidson, and Schwarz, 1999; Wong, C., 1997; Nelly and Weller, 1998).

The contrarian trading rules (BBs) generated average profits of 2.19%, 3.37%, and 2.52% for all thirty stocks with daily data. The BBs generated profits on 23 of the 30 stocks with daily data, with the most profitable being American Express (AXP), and the least profitable being IBM. The results with weekly data are more volatile with average profits of 15.5%, 15.2%, and 17.0% from the trading rules, with only 21 of the 30 stocks being profitable.

5.2 – Hurst exponent and profits from trending trading rules (Hypotheses 1)

Hypothesis 1 postulates that stocks with higher H should yielder high profits from trending trading rules. To test this relation, each DJIA component was grouped according to its H. The first group includes stocks with a H less than 0.5. The second group includes stocks with a H that is greater than 0.5 but less than 0.55. The final group includes all stocks with a H that is greater than 0.55.

Table 4 presents the results of the classification analysis in the form of a contingency table. Panel A presents the results for the combined weekly and daily data, while Panel B and Panel C present the results for daily and weekly data, respectively.

Insert Table 4 about here

The results provide strong evidence that profits from trending trading rules are partially explained by the long-term dependencies, as identified by the *H* estimation. All three Panels reveal that profits are lowest for stocks with *H* that are less than 0.5 and increase in association with an increasing *H*. The trading rules earned returns of 11.5% in excess of the buy-and-hold strategy for stocks with a *H* greater than 0.55, while technical analysis underperformed by 16.7% for stocks with *H* less than 0.5.

The robustness of the results is tested though sub-period analysis. Table 5 presents the estimation of the *H* over three sub-periods (n.b. the sub-periods divide the data set into three equal periods). The profits from the technical trading rules are also calculated on the same sub periods (untabulated).

Insert Table 5 about here

The results of the sub-period analysis for the association between the *H* and the profits to technical analysis are presented in Table 6. The sub-period analysis confirms the robustness of the results in Table 4 as profits are higher on the stocks that exhibit long-term dependencies as identified by the *H*. Sub-period 1 and 3 exhibit consistent patterns of increasing returns in conjunction with increasing *H;* however, the weekly data on sub-period 2 does not reflect the synthesis.

Insert Table 6 about here

These results are consistent with the synthesis and tend to corroborate the proposition that *H* less than 0.5 exhibit anti-persistent trends that limit trending trading rules' ability to identify patterns, whereas time series with a *H* greater than 0.55 exhibit persistent trends that are identified by the trending trading rules to earn profits. However, the results partially explain, as opposed to completely explain, the profits from the trading rules because there are anomalies. For example, Pfizer's weekly time series exhibited an anti-persistent nature (*H* of 0.431), yet technical analysis was able to earn an average profit of 14.2%.

The tests in Table 4 and Table 6 provide evidence to support hypothesis one; however, the classification system does not offer the precision of statistical rigour. Therefore, additional tests of the relationship between the *H* and profits from technical analysis are provided through regression analysis. The results of the regression of Equation 12 are presented in Table 7. Panel A presents the estimation using average returns for all three trading rules as a proxy for $Profit_i$, while Panel B presents the results of the estimation using the average of each trading rule (MACO, filter, and TRBO) as a proxy for $Profit_i$.

Insert Table 7 about here

The results are consistent with, and corroborate, the results presented in Tables 3 and 5, and further support the synthesis in hypothesis one: the *H* is able to identify long-term dependencies in time series data that are exploited by technical trading rules to generate profits. The estimation has an R^2 of 31%, with virtually all of the explanatory power resulting from the *H* variable. Panel B presents additional insight, revealing the resulting R^2 of 37% when using the MACO and TRBO proxies for profits. The estimation with the filter rule as a proxy for profits do not yield strong results. This is a function of the aforementioned lack of profitability for filter rules. The sub-period analysis conducted in Section 5.3 provides further data to test the robustness of the estimation in Equation 12. The results of the estimation with the sub-period data are presented in Table 8.

Insert Table 8 about here

The sub-period estimation, with the daily data, results in an R^2 of 0.334 to corroborate the estimation results of equation 12. The estimation results with weekly data reveal a weaker relationship (R^2 of 0.084). Overall, the empirical evidence provides strong support for the acceptance of Hypothesis 1 as trending trading rules are more profitable on stock with higher long-term dependencies.

5.3 – Hurst exponent and profits from contrarian trading rules (Hypotheses 2)

Hypotheses 2 postulates that stocks with lower *H* should yielder high profits from contrarian trading rules. To test this relation between the *H* and profits from the contrarian trading rules, each DJIA component was grouped according to its *H*. The groupings are the same as in the test of Hypothesis

1. Table 9 presents the results of the classification analysis for the contrarian trading rules. Panel A presents the results for the combined weekly and daily data, while Panel B and Panel C present the results for daily and weekly data, respectively.

Insert Table 9 about here

The results provide evidence that profits from contrarian trading rules are partially explained by the long-term anti-dependencies in a time series. Panel A reveals that profits are highest for stocks with *H* that are less than 0.5 and decrease in association with an increasing *H*. Contrarian trading rules were able to earn returns of 11.3% in excess of the buy-and-hold trading strategy for stocks with a *H* less than 0.5. The results from Panel B (daily data) and Panel C (weekly data) are not as consistent.

Again, regression analysis between the *H* and profits from contrarian rules is conducted. The results of the regression of Equation 12 are presented in Table 10. The results are consistent with Tables 9 as the regression provides some evidence of the synthesis in hypothesis two. The estimation has a low R^2 of 5%, however, the intercept for the *H* variable is negative and significant at the 10% level.

Insert Table 10 about here

Overall, the empirical evidence provide some support for the acceptance of Hypothesis 2 as contrarian trading rules appear to be more profitable on stock with lower *H*.

5.4 – Lagged H and the profits from technical analysis (Hypothesis 3)

Hypothesis 3 was proposed to determine if investors can use the *H* in one-period to predict which stocks will provide the most profits from technical analysis in the following period. A test is conducted through OLS regression between the profits from the trending trading rules in sub-period 2 and 3 and the lagged *H* (sub period 1 and 2, respectively). For example, can Caterpillars' *H* of 0.587 in the first sub-period be used to forecast profits through a trending trading rule for Caterpillar in the second period? If so, then investors can use information about a time series dependencies to earn profit. The results of the regression estimation are presented in Table 11 (Panel A).

Insert Table 11

The estimation results suggest that the lagged *H* is not able to forecast future profits on a time series. These results are influence by the *H* instability across sub-periods. This has important implications for investors. If the *H* is constantly changing for a time series, investor will be required to make subjective decisions and forecasts to develop expectation of a time series long-term dependencies to earn profits from technical analysis. Corazza and Malliaris (2002) also note that the *H* is not fixed but changes dynamically over time.

In order to mitigate the impacts that instable an *H* can have on forecasting profits in future periods, an additional exploratory test is proposed. The same OLS regression proposed regarding the lagged *H* was conducted with the daily and weekly time series that exhibit the lowest standard deviation of the *H* across the sub-period. The low standard deviation is used as a proxy for a stable *H*. The results are presented in Table 11 – Panel B. The estimation results with both weekly and daily data are consistent with Table 11 – Panel A, rejecting Hypothesis 3 by suggesting that the lagged *H* is not able to forecast future profits on a time series. However, there does appear to be some predictive ability for traders as the estimation that utilizes only the daily data result in an R^2 of 0.213, suggesting that there may be profit potential with daily data.

To obtain additional insight into the nature of the *H* across sub-periods, regression estimation is performed on the standard deviation of the *H* (regressant) across sub periods for a given time series with the standard deviation (regressar) of the corresponding time series to investigate any such relationship. The results of the regression are presented in Table 12.

Insert Table 12 about here

The results reveal that there is a strong, negative relationship between the standard deviation of a data set and the resulting standard deviation of the *H* calculated on sub-periods of the data set. Stated differently, the estimated *H* on sub-periods of a time series appears to be more stable with time series that have higher variance. However, these empirical results must also consider that the variance of a data set increases with its scale (i.e. weekly data has a higher σ than daily data). Therefore, the results can also be interpreted to suggest that the *H* is less volatile on the weekly data.

5.5 – Profits from technical analysis on different scales (Hypothesis 4)

Table 13 presents comparative summary statistics between profits from the trading rules calculated at different scales (daily and weekly). Overall, the average excess return for all trading rules is negative 2.58%. Moving averages results in negative 2.24% profits, while filter rules, Bollinger Bands and trading range break-out rules resulted in negative 10.59%, 2.7% and negative 0.22% profit, respectively. Calculating the trading signals with weekly data resulted in an average profit for all trading rules of 0.25%. Moving averages results in negative 3.10% profits, while filter rules, Bollinger Bands and trading range break-out rules resulted in negative 22.6%, positive 15.9% and positive 10.8% profits, respectively.

Insert Table 13 about here

Table 13 reveals that the trading rules were more profitable when calculated with weekly data as 201 of the 360 variants (30 stocks x 4 trading rules x 3 variants of each rule) were profitable, or 55.8%, while only 161, or 44.7%, variants were profitable with daily data. As discussed earlier, all stocks that resulted in profits from technical analysis when calculated with daily data are also profitable with weekly data. However, the weekly data resulted in much more variation in the profits made evident by wider ranges and higher standard deviations.

The empirical evidence reveals that technical trading rules result in different profit levels when calculated at different scales, and therefore rejects the Hypothesis 4.

6. Conclusion

This paper works towards a synthesis between technical analysis and fractal geometry. The Hurst exponent (H) was developed from the field of fractal geometry and provides a statistical technique to identify the nature of any dependencies in a time series. Technical analysis has developed various trading rules that are premised on the belief that past price data reveals patterns that can be used to predict future prices. Based on this logic, there is a natural synthesis that suggests that time series with high H should result in higher profits trending trading rules and time series with low H should result in higher profits from contrarian trading rules. This paper develops and empirically tests this synthesis.

Currently, much research is being conducted on capital markets and technical analysis. Research on the application of fractal geometry to capital markets has been much more limited. More specifically, there are no known studies that investigate the relationship between technical analysis and fractal geometry. The extant literature solely investigates the *H's* ability to identify financial market predictability. This paper makes a significant contribution by extending the literature to determine if technical analysis is able generate abnormal returns (after accounting for transaction costs) on time series that exhibit long term dependencies or anti-dependence. The paper also provides a comprehensive examination of moving averages cross-over, filter, Bollinger Band and trading ranges break-out rules on the DJIA components at different scales.

Two tests are conducted to evaluate the relationship between the H and profits to technical analysis. Firstly, the financial series are classified into three groups based on their H ($H<0.5$; $0.5<H>0.55$; $H>0.55$) to determine if time series with a higher (lower) H results in higher returns to trending (contrarian) trading rules. Secondly, statistical tests of the relationship between the H and profits to technical analysis are estimated through OLS regression. Both tests provide evidence that the H is able to identify long-term dependencies and anti-dependence that result in higher (lower) profits to trending (contrarian) trading rules. Therefore, profits from trending (contrarian) trading rules are higher for time series that exhibit long-term dependencies (anti-dependence). This is consistent with the main postulate of the synthesis (Hypotheses 1 and 2).

The substantial data requirements skew the sample towards the larger Dow Jones component firms. Therefore, the results may not necessarily be generalizable to a broader population that includes smaller firms. Additional testing should be conducted using a more diverse sample of firms (e.g. all 500 stocks of the S&P 500) and for a longer period of time. The data sets limitation may also impact the empirical test for determining whether investors can use the H in one-period to predict which stocks will provide the most profits from technical analysis in the next period (Hypothesis 3).

There are a many future research opportunities to further develop the synthesis between fractal geometry and technical analysis. The first priority is to further test the relationship between profits and H with a more robust data, likely making use of global equity markets and various firm sizes. Utilizing time series from the global equity market will provide future researchers with a larger range of H in their sample and also provide access to a larger pool of securities. Researchers should also seek to understand what causes anomalies in the synthesis. For example, Pfizer's weekly time series

exhibited an anti-persistent nature (*H* of 0.431), yet technical analysis was able to earn an average profit of 14.2%. Researchers should investigate whether this, and other anomalies, is the result of technical analysis ability utilize past price data to develop a trading signal that is more powerful than the data alone.

Finally, and most importantly for investors, researchers should continue to investigate how the *H* in one sub period can be used, ex ante, to identify which time series will yield the most fruitful results from technical analysis. This paper has made a contribution in this area by suggesting that the sub-periods with daily data are able to predict future sub-period profits from technical trading rules. Researchers are encouraged to continue research in this area to provide a significant impact to traders and investors.

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Tables

Table 1 - Sample data and descriptive statistics (July 22, 1998 to July 22, 2008)

		Daily series (n = 2,514)				Weekly series (n = 520)			
Company	symbol	r	σ	skew	kur	r	σ	skew	Kur
Alcoa	AA	.014%	1.02%	.189	2.189	.070%	2.19%	-.012	1.057
American International Group	AIG	-.010%	0.88%	.032	3.950	-.056%	1.94%	.179	2.686
American Express	AXP	.006%	0.97%	-.057	3.138	.033%	2.00%	-.486	3.914
Boeing	BA	.009%	0.91%	-.707	7.998	.057%	2.08%	-1.438	11.56
Bank of America	BAC	.000%	0.89%	.512	7.752	.001%	1.95%	.297	4.104
Citigroup	C	-.004%	0.97%	.056	4.746	-.017%	2.10%	.050	2.505
Caterpillar	CAT	.022%	0.90%	-.173	3.994	.108%	2.02%	.376	2.552
Chevron Corporation	CVX	.018%	0.67%	-.057	1.415	.087%	1.34%	-.367	.225
DuPont	DD	-.001%	0.79%	.112	3.117	-.001%	1.74%	-.137	1.757
Walt Disney	DIS	-.002%	0.94%	-.179	7.735	-.007%	1.92%	-.466	4.833
General Electric	GE	.002%	0.80%	-.081	4.906	.012%	1.67%	-.194	4.376
General Motors	GM	-.019%	1.08%	.071	4.276	-.091%	2.40%	-.119	2.705
Home Depot	HD	-.003%	1.02%	-1.169	19.46	-.012%	2.29%	-.794	5.615
Hewlett-Packard	HPQ	.014%	1.18%	-.085	5.236	.079%	2.43%	.027	1.437
IBM	IBM	.014%	0.86%	-.178	7.691	.069%	1.81%	-.026	2.519
Intel	INTC	.003%	1.26%	-.457	6.207	.012%	2.53%	-.621	3.014
Johnson & Johnson	JNJ	.013%	0.62%	-.634	11.64	.063%	1.34%	.148	5.312
JPMorgan Chase	JPM	.002%	1.04%	.352	5.790	.012%	2.25%	.165	2.243
Coca-Cola	KO	-.006%	0.69%	-.238	5.581	-.027%	1.53%	-.519	5.723
McDonalds	MCD	.013%	0.79%	-.082	4.820	.062%	1.61%	-.151	1.172
3M	MMM	.013%	0.68%	.136	3.817	.067%	1.47%	.051	2.360
Merck	MRK	-.004%	0.84%	-2.026	31.19	-.014%	1.84%	-.760	4.481
Microsoft	MSFT	.001%	0.96%	-.176	6.789	.006%	1.98%	.081	2.970
Pfizer	PFE	-.009%	0.81%	-.324	3.785	-.042%	1.71%	-.243	2.084
Proctor & Gamble	PG	.010%	0.73%	-4.200	89.08	.048%	1.69%	-4.411	53.05
AT&T	T	.002%	0.86%	-.070	3.205	.009%	1.81%	.315	2.920
United Technologies Corporation	UTX	.019%	0.84%	-1.628	26.18	.096%	1.84%	-2.298	23.02
Verizon Communication	VZ	.003%	0.82%	0.059	3.799	.015%	1.62%	.098	2.130
Wal-Mart	WMT	.011%	0.83%	2.919	.106	.058%	1.75%	.179	2.067
ExxonMobil	XOM	.018%	0.68%	-.055	2.106	.008%	1.35%	-.289	1.117

r – the mean log daily return

σ - the standard deviation of the log daily returns

skew – skewness of the raw price data

kur – kurtosis of the raw price data

Table 2: Hurst exponent and technical trading rule profits with daily data.

Panel A - Daily data results (in percentage) for trending trading rules

Symbol	*H*	MACO (1,50)	MACO (1,200)	MACO (5,150)	Filter (1%)	Filter (2%)	Filter (5%)	TRBO (50)	TRBO (150)	TRBO (200)	Average
AA	0.501	-3.4	-11.4	0.3	-28.7	-20.1	-7.7	-4.9	-7.5	-8.0	-10.16
AIG	0.558	8.9*	12.2*	15.2*	12.7*	-1.6	2.3	10.5*	4.7	8.9*	**8.20**
AXP	0.488	-13.8	-6.9	-3.1	-19.5	-17.4	-12.0	-4.8	3.5	2.5	-7.94
BA	0.542	1.5	8.8*	5.1	-14.7	-6.5	-11.9	0.7	2.9	8.5*	-0.62
BAC	0.549	-5.9	-2.7	-2.9	0.9	-6.7	3.0	-12.7	2.5	2.4	-2.46
C	0.587	1.3	3.2	0.2	1.7	-13.0	1.9	-3.7	14.4*	14.1*	**2.23**
CAT	0.512	-19.9	-11.0	-7.1	-18.6	-21.2	2.6	-13.3	-6.2	-2.9	-10.84
CVX	0.479	-20.4	-17.1	-13.1	-33.8	-19.4	-10.3	-13.0	-5.6	2.4	-14.48
DD	0.463	-7.6	-6.4	-7.0	-10.6	-13.6	-0.5	-8.9	-7.5	-5.5	-7.51
DIS	0.497	-8.0	5.5*	4.1	-10.3	-17.6	-7.3	6.1	9.3*	3.5	-1.63
GE	0.519	-1.8	7.2*	4.1	-9.0	-13.8	-3.6	-8.4	-14.2	9.1*	-3.38
GM	0.549	17.0*	18.1*	21.3*	7.5	21.9*	15.6*	17.4*	15.7*	8.7	**15.91**
HD	0.507	1.2	10.3*	5.3	-3.4	-13.9	7.4	-0.6	2.9	5.2	**1.60**
HPQ	0.564	-4.7	11.4*	12.4*	-24.4	-11.5	-5.0	12.8	7.4	11.0*	**1.04**
IBM	0.540	3.1	-7.6	-2.5	-17.6	-19.6	-3.4	-11.7	-8.7	-3.7	-7.97
INTC	0.542	4.6	16.3*	19.0*	-14.2	-14.3	1.4	9.7	13.4*	2.8	**4.30**
JNJ	0.464	-13.9	-6.5	-3.4	-12.3	-24.1	-7.6	-8.8	-4.9	-7.9	-9.93
JPM	0.530	-2.0	-5.4	-6.8	-5.9	-6.8	-7.8	-15.0	4.0	5.2	-4.50
KO	0.518	-3.0	-3.2	-1.8	-5.4	-7.3	6.2*	-0.4	3.6	0.8	-1.17
MCD	0.573	-4.1	-2.2	-1.6	-18.1	-23.4	-5.6	-2.3	0.9	6.2*	-5.58
MMM	0.464	-14.0	-12.1	-12.3	-22.9	-21.1	-15.2	-9.5	-10.8	-9.4	-14.14
MRK	0.541	-0.4	8.8*	9.7*	-9.6	-10.8	-9.8	4.0	6.1	13.5*	**1.28**
MSFT	0.507	-2.3	-1.2	0.9	-18.1	-12.7	-1.2	-4.2	-1.7	-0.6	-4.57
PFE	0.452	-1.7	-1.9	-1.2	-7.1	-19.3	-5.0	3.2	5.3	10.1*	-1.96
PG	0.500	-7.6	-6.6	0.9	-20.1	-17.8	1.8	-1.8	-6.9	-4.6	-6.97
T	0.512	-1.2	3.1	-6.3	-22.5	-7.7	0.3	-1.7	4.1	6.0*	-2.88
UTX	0.491	-13.9	-10.7	-7.1	-26.0	-23.0	-2.8	-11.9	-5.0	0.4	-11.11
VZ	0.503	-7.7	0.8	1.3	-25.0	-8.6	-1.4	3.2	1.1	6.6*	-3.30
WMT	0.486	-16.1	-16.4	-15.9	-16.1	-31.4	-10.2	-14.2	-7.8	-8.0	-15.12
XOM	0.477	-21.9	-15.6	-12.1	-33.9	-29.3	-10.5	-17.3	-7.0	-4.0	-16.84
Average		-5.26	-1.31	-0.15	-14.17	-14.39	-3.21	-3.38	**0.27**	**2.44**	

*bootstrap simulation p-value is significant at both the 5% and 10% level of significance

Row average is the average of all nine trading rule variants

Column average is the average profit for the individual trading rule variant

All positive profits are in bold

Table 2: Hurst exponent and technical trading rule profits with daily data.

Panel B - Daily data results (in percentage) for contrarian trading rules

Symbol	*H*	BB (20,2)	BB (20,1)	BB (30,2)	Average
AA	0.501	-1.0	1.9	7.5	**2.80**
AIG	0.558	7.7	14.0	-1.8	**6.63**
AXP	0.488	12.9	16.9	10.2	**13.33**
BA	0.542	-8.5	-2.7	-9.6	-6.93
BAC	0.549	5.4	2.4	5.1	**4.30**
C	0.587	4.2	0.0	1.3	**1.83**
CAT	0.512	-0.3	5.1	-1.7	**1.03**
CVX	0.479	-0.4	-8.2	4.9	-1.23
DD	0.463	12.6	10.5	5.5	**9.53**
DIS	0.497	7.7	13.7	6.0	**9.13**
GE	0.519	3.4	5.3	-1.2	**2.50**
GM	0.549	-0.6	-6.5	0.2	-2.30
HD	0.507	2.2	8.5	4.4	**5.03**
HPQ	0.564	3.9	9.7	0.6	**4.73**
IBM	0.540	-11.8	-15.4	-2.0	-9.73
INTC	0.542	-3.8	2.8	2.4	**0.47**
JNJ	0.464	5.7	2.0	4.3	**4.00**
JPM	0.530	-1.6	2.0	0.7	**0.37**
KO	0.518	8.2	3.9	7.9	**6.67**
MCD	0.573	-2.5	-5.0	-2.1	-3.20
MMM	0.464	0.7	2.5	0.9	**1.37**
MRK	0.541	0.9	2.7	2.4	**2.00**
MSFT	0.507	4.4	0.2	6.3	**3.63**
PFE	0.452	10.7	9.8	10.7	**10.40**
PG	0.500	-5.6	0.7	-2.0	-2.30
T	0.512	2.0	3.5	6.6	**4.03**
UTX	0.491	-4.2	2.3	-8.9	-3.60
VZ	0.503	6.1	2.4	6.4	**4.97**
WMT	0.486	2.7	15.5	6.2	**8.13**
XOM	0.477	4.7	0.5	4.4	**3.20**
Average		**2.19**	**3.37**	**2.52**	

*bootstrap simulation p-value is significant at both the 5% and 10% level of significance
Row average is the average of all three trading rule variants
Column average is the average profit for the individual trading rule variant
All positive profits are in bold

Table 3: Hurst exponent and technical trading rule profits with weekly data.

Panel A - Weekly Data results (in percentage) for trending trading rules

Symbol	*H*	MACO (1,50)	MACO (1,200)	MACO (5,150)	Filter (1%)	Filter (2%)	Filter (5%)	TRBO (50)	TRBO (150)	TRBO (200)	Average
AA	0.498	-46.3	-41.3	-18.5	-17.3	-57.4	-4.3	-24.7	30.6	10.6	-18.7
AIG	0.597	47.2*	30.7*	22.9*	-15.0	19.5	3.2	20.6	38.8*	77.8*	**27.3**
AXP	0.497	-4.1	27.1*	34.1*	-50.2	32.8*	-52.6	-0.9	4.8	-26.8	-4.0
BA	0.543	42.8*	-31.6	-23.1	-33.4	5.2	-10.6	31.4*	45.7*	-38.9	-1.4
BAC	0.562	-7.5	39.7*	63.4*	-9.2	6.3	12.5	56.0*	21.6	25.4	**23.1**
C	0.603	17.2	67.7*	40.2*	-24.3	24.8*	4.6	48.7*	48.7*	28.0*	**28.4**
CAT	0.522	-54.0	-51.4	-42.1	-99	-64.9	-82.7	16.4	-15.1	-54.6	-49.7
CVX	0.488	-69.3	-12.2	-38.8	-90.0	-43.2	-82.9	18.6	9.9	-11.9	-35.5
DD	0.463	-33.0	-31.5	-25.0	-24.0	-27.2	-17.9	-35.3	6.1	2.6	-20.6
DIS	0.503	16.2*	-38.3	47.7*	-23.1	-22.2	-15.0	0.0	34.1*	-14.5	-1.7
GE	0.556	19.1	1.9	23.5*	-35.3	-1.3	-24.4	39.9*	28.6*	-17.5	**3.8**
GM	0.554	36.7*	24.8	23.2	38.6*	41.1*	78.3*	18.0	25.8*	90.4*	**41.9**
HD	0.524	11.9	25.8*	6.7	-16.6	-8.2	13.4	35.8*	35.8*	61.8*	**18.5**
HPQ	0.564	46.2*	9.2	58.3*	-56.2	-21.8	-55.7	27.3*	50.0*	-29.2	**3.1**
IBM	0.566	-31.0	-28.4	-24.2	-33.6	34.9*	19.1	-14.6	36.7*	-12.9	-6.0
INTC	0.570	64.5*	-24.6	5.1	-12.7	20.0	46.7*	-6.6	30.9*	46.7*	**18.9**
JNJ	0.451	-34.8	-45.9	-14.2	-77.3	-62.4	-55.5	-38.7	-3.6	5.1	-36.4
JPM	0.523	-17.2	-27.8	17.7	-7.2	-3.7	-1.6	27.4*	14.5	-10.7	-1.0
KO	0.518	-22.7	2.2	-26.4	-8.8	-17.9	7.8	7.2	-6.2	13.7	-5.7
MCD	0.629	33.5*	38.4*	64.6*	-26.6	-63.3	-60.6	27.2*	15.9	6.6	**4.0**
MMM	0.445	-61.8	-21.7	-24.5	-80.6	-12.3	-44.3	-31.1	1.0	-4.3	-31.1
MRK	0.566	15.2	10.5	34.1*	-8.2	-11.4	-6.0	23.8	33.6*	8.3	**11.1**
MSFT	0.506	3.0	-11.5	13.7	22.7*	13.9	-6.8	12.5	25.0*	-10.5	**6.9**
PFE	0.431	-5.4	18.1	9.2	-20.4	-9.5	2.6	17.9	61.0*	54.2*	**14.2**
PG	0.484	-23.9	-20.2	-36.1	-55.6	-5.8	-13.6	-36.9	-59.9	-17.1	-29.9
T	0.532	12.8	28.2	52.8*	-49.0	-23.5	-26.8	13.2	46.9*	20.1	**8.3**
UTX	0.478	-46.6	-47.9	-47.4	89.1	-55.7	-24.8	-20.2	22.0	-34.7	-18.5
VZ	0.514	-7.5	3.1	15.8	-20.7	-6.1	-47.3	-12.2	23.2*	5.8	-5.1
WMT	0.505	-59.5	-42.8	-44.8	-91.2	-50.7	-77.1	-28.2	11.9	8.8	-41.5
XOM	0.474	-74.4	-32.8	-36.1	-146.5	-70.9	-86.6	-11.0	12.8	-22.4	-52.0
Average		-7.8	-6.1	**4.4**	-32.7	-14.7	-20.3	**6.1**	**21.0**	**5.3**	

*bootstrap simulation p-value is significant at both the 5% and 10% level of significance
Row average is the average of all nine trading rule variants
Column average is the average profit for the individual trading rule variant
All positive profits are in bold

Panel B - Weekly Data results (in percentage) for contrarian trading rules

Symbol	*H*	BB (20,2)	BB (20,1)	BB (30,2)	Average
AA	0.498	9.5	3.8	22.0	**11.77**
AIG	0.597	8.0	26.7	4.1	**12.93**
AXP	0.497	4.9	23.2	4.9	**11.00**
BA	0.543	-34.0	-45.3	-54.1	-44.47
BAC	0.562	24.4	32.2	16.5	**24.37**
C	0.603	15.1	29.0	-6.3	**12.60**
CAT	0.522	-51.9	-62.3	-22.2	-45.47
CVX	0.488	-8.8	-1.6	-9.2	-6.53
DD	0.463	67.0	61.1	97.4	75.17
DIS	0.503	-9.0	-6.2	-8.4	-7.87
GE	0.556	-16.8	7.0	-3.2	-4.33
GM	0.554	15.7	12.8	8.4	**12.30**
HD	0.524	11.5	36.5	22.3	**23.43**
HPQ	0.564	-26.3	-31.3	-13.2	-23.60
IBM	0.566	52.6	19.7	71.9	**48.07**
INTC	0.570	-1.7	7.3	22.0	**9.20**
JNJ	0.451	28.5	7.2	56.2	**30.63**
JPM	0.523	47.5	71.0	47.2	**55.23**
KO	0.518	30.5	25.1	2.8	**19.47**
MCD	0.629	-19.6	-13.0	-38.0	-23.53
MMM	0.445	15.7	26.2	43.8	**28.57**
MRK	0.566	2.3	1.4	-2.0	**0.57**
MSFT	0.506	71.5	42.4	71.4	**61.77**
PFE	0.431	46.0	37.5	48.3	**43.93**
PG	0.484	11.1	-20.8	4.4	-1.77
T	0.532	65.6	11.3	27.7	**34.87**
UTX	0.478	-8.1	35.4	-26.2	**0.37**
VZ	0.514	25.9	19.1	37.7	**27.57**
WMT	0.505	95.2	117.1	122.5	**111.60**
XOM	0.474	-8.6	-17.6	-36.9	-21.03
Average		**15.46**	**15.16**	**17.06**	

*bootstrap simulation p-value is significant at both the 5% and 10% level of significance
Row average is the average of all three trading rule variants
Column average is the average profit for the individual trading rule variant
All positive profits are in bold

Table 4 - Returns to trending trading rules classified by *H*

Panel A – Profits from the average of all trading rules (daily and weekly)

H grouping	Total (%)	MACO (%)	Filter (%)	TRBO (%)	N
0.5 > *H*	-16.791	-18.549	-27.986	-3.826	19
0.5 < *H* > 0.55	-4.529	-2.490	-14.680	3.593	27
0.55 < *H*	11.535	18.436	-4.762	20.938	14

Panel B – Profits from the average of all trading rules (daily)

H grouping	Total (%)	MACO (%)	Filter (%)	TRBO (%)	N
0.5 > *H*	-9.613	-9.337	-15.959	-3.548	9
0.5 < *H* > 0.55	-2.934	-0.027	-8.588	-0.182	17
0.55 < *H*	1.4725	4.350	-7.000	7.075	4

Panel C – Profits from the average of all trading rules (weekly)

H grouping	Total (%)	MACO (%)	Filter (%)	TRBO (%)	N
0.5 > *H*	-23.250	-26.840	-38.810	-4.077	10
0.5 < *H* > 0.55	-7.240	-6.677	-25.037	10.010	10
0.55 < *H*	15.56	24.070	-3.867	21.910	10

Table 5 - Hurst exponent estimation on sub-periods

	Daily Data						Weekly Data				
Symbol	H	H_1	H_2	H_3	σ_H		H	H_1	H_2	H_3	σ_H
AA	0.501	.511	.556	.563	0.0282		0.498	.505	.615	.595	0.0586
AIG	0.558	.550	.531	.620	0.0469		0.597	.593	.514	.689	0.0876
AXP	0.488	.545	.514	.497	0.0243		0.497	.592	.487	.533	0.0526
BA	0.542	.573	.589	.571	0.0099		0.543	.536	.654	.605	0.0593
BAC	0.549	.451	.514	.616	0.0833		0.562	.426	.486	.629	0.1043
C	0.587	.577	.552	.612	0.0301		0.603	.609	.612	.618	0.0046
CAT	0.512	.511	.563	.525	0.0269		0.522	.524	.604	.540	0.0423
CVX	0.479	.431	.537	.485	0.0530		0.488	.412	.582	.476	0.0859
DD	0.463	.461	.453	.527	0.0406		0.463	.454	.424	.598	0.0930
DIS	0.497	.524	.521	.524	0.0017		0.503	.559	.527	.592	0.0325
GE	0.519	.533	.517	.526	0.0080		0.556	.566	.531	.593	0.0311
GM	0.549	.568	.585	.590	0.0115		0.554	.582	.595	.652	0.0372
HD	0.507	.493	.582	.500	0.0495		0.524	.493	.647	.517	0.0829
HPQ	0.564	.576	.537	.460	0.0590		0.564	.561	.691	.429	0.1310
IBM	0.540	.512	.587	.514	0.0427		0.566	.555	.621	.498	0.0616
INTC	0.542	.557	.578	.535	0.0215		0.570	.630	.665	.515	0.0785
JNJ	0.464	.497	.522	.477	0.0225		0.451	.551	.518	.468	0.0418
JPM	0.530	.520	.600	.530	0.0436		0.523	.502	.665	.481	0.1007
KO	0.518	.462	.543	.586	0.0630		0.518	.448	.539	.700	0.1276
MCD	0.573	.494	.631	.510	0.0749		0.629	.527	.760	.498	0.1436
MMM	0.464	.442	.442	.553	0.0641		0.445	.441	.412	.592	0.0966
MRK	0.541	.473	.531	.619	0.0735		0.566	.431	.509	.737	0.1590
MSFT	0.507	.547	.483	.561	0.0416		0.506	.552	.450	.608	0.0801
PFE	0.452	.470	.550	.533	0.0421		0.431	.439	.621	.559	0.0925
PG	0.500	.541	.458	.509	0.0419		0.484	.535	.453	.513	0.0424
T	0.512	.451	.506	.533	0.0418		0.532	.410	.565	.586	0.0961
UTX	0.491	.531	.547	.506	0.0207		0.478	.530	.590	.509	0.0420
VZ	0.503	.479	.555	.556	0.0442		0.514	.472	.607	.615	0.0804
WMT	0.486	.488	.469	.488	0.0110		0.505	.505	.436	.478	0.0348
XOM	0.477	.416	.530	.483	0.0573		0.474	.396	.578	.490	0.0910

H_1 – Hurst exponent for the first sub period of 07/22/1998 to 11/19/2001

H_2 – Hurst exponent for the second sub period of 11/20/2001 to 03/21/2005

H_3 – Hurst exponent for the third sub period of 03/22/2005 to 07/21/2008

σ_H – standard deviation of the H across the three sub periods.

Table 6 - Returns to technical trading rules classified by *H* on sub periods

Panel A – Sub-period 1 (07/22/1998 to 11/19/2001)

H grouping	Total (%)	MACO (%)	Filter (%)	TRBO (%)	N
I. Profits from the average of daily and weekly trading rule returns					
0.5 > *H*	-0.170	-0.195	-0.279	0.009	25
0.5 < *H* > 0.55	-0.098	-0.061	-0.213	0.015	19
0.55 < *H*	0.067	0.192	-0.125	0.232	16
II. Profits from the average of daily trading rule returns					
0.5 > *H*	-0.142	-0.146	-0.178	-0.102	14
0.5 < *H* > 0.55	-0.081	-0.043	-0.163	-0.037	11
0.55 < *H*	-0.003	0.065	-0.080	0.047	5
III. Profits from the average of weekly trading rule returns					
0.5 > *H*	-0.205	-0.258	-0.407	0.151	11
0.5 < *H* > 0.55	-0.121	-0.086	-0.282	0.086	8
0.55 < *H*	0.099	0.250	-0.146	0.315	11

Panel B – Sub-period 2 (11/20/2001 to 03/21/2005)

H grouping	Total (%)	MACO (%)	Filter (%)	TRBO (%)	N
I. Profits from the average of daily and weekly trading rule returns					
0.5 > *H*	-0.329	-0.290	-0.568	-0.129	12
0.5 < *H* > 0.55	-0.015	-0.019	-0.105	0.065	19
0.55 < *H*	-0.053	-0.039	-0.172	0.070	29
II. Profits from the average of daily trading rule returns					
0.5 > *H*	-0.103	-0.116	-0.136	-0.059	5
0.5 < *H* > 0.55	-0.039	-0.030	-0.092	-0.013	13
0.55 < *H*	0.037	0.085	-0.061	0.131	12
III. Profits from the average of weekly trading rule returns					
0.5 > *H*	-0.490	-0.415	-0.877	-0.179	7
0.5 < *H* > 0.55	0.035	0.003	-0.132	0.233	6
0.55 < *H*	-0.117	-0.126	-0.251	0.028	17

Panel C – Sub-period 3 (03/22/2005 to 07/21/2008)

H grouping	Total (%)	MACO (%)	Filter (%)	TRBO (%)	N
I. Profits from the average of daily and weekly trading rule returns					
0.5 > *H*	-0.348	-0.237	-0.643	-0.131	14
0.5 < *H* > 0.55	-0.096	-0.057	-0.261	0.007	19
0.55 < *H*	0.021	0.067	-0.041	0.037	27
II. Profits from the average of daily trading rule returns					
0.5 > *H*	-0.119	-0.124	-0.175	-0.058	6
0.5 < *H* > 0.55	-0.060	-0.052	-0.099	-0.028	13
0.55 < *H*	0.070	0.095	0.022	0.093	11
III. Profits from the average of weekly trading rule returns					
0.5 > *H*	-0.519	-0.323	-0.994	-0.186	8
0.5 < *H* > 0.55	-0.174	-0.067	-0.612	0.084	6
0.55 < *H*	-0.013	0.048	-0.084	-0.002	16

Table 7 - Estimation results from regression of profits from trending trading rules on *H*.

This table presents the results of the regression between the profits generated by technical analysis on a time series and long-term dependencies (*H* as proxy), as expressed by the following equation:

$$\text{TTR Profits} = a_o + a_1\text{Hurst Exponent}(H) + \varepsilon_1$$

Panel A – Profits from the average of all trading rules

Estimation	Intercept	Hurst	Adjusted R^2	F-value	N
1 – *H*	-127.82 (-5.43)	237.75* (5.24)	0.310	27.55	60

Hurst is the nature of the dependencies in the time series as measured by the Hurst exponent
*estimation is significant at both the 5% and 1% level of significance

Panel B – Average profits from the moving average, filter, and trading range break out rules

Estimation	Intercept	Hurst	Adjusted R^2	F-value	N
1 – MACO	-176.34 (-6.12)	334.82* (5.81)	0.372	18.51	60
2 – Filter	-110.50 (-2.88)	190.93* (2.48)	0.066	3.09	60
3 – TRBO	-91.67 (-4.33)	157.67* (3.72)	0.370	18.33	60

Hurst is the nature of the dependencies in the time series as measured by the Hurst exponent
*estimation is significant at both the 5% and 1% level of significance

Table 8 - Estimation results from regression of profits from trending trading rules on *H* with sub-period data

This table presents the results of the regression between the profits generated by technical analysis on a time series and the time series' long-term dependencies (*H* as proxy) by using the sub-period data, as expressed by the following equation:

$$\text{TTR Profits}_1 = a_o + a_2\text{Hurst Exponent}(H)_1 + \varepsilon_1$$

Estimation	Intercept	Hurst Exponent	Adjusted R^2	F-value	N
1 – daily and weekly	-67.48 (-3.20)	112.54 (2.88)	0.039	0.004	180
2 – daily	-0.84 (7.12)	1.51* (6.75)	0.334	45.67	90
3 – weekly	-111.69 (-3.23)	178.23* (2.85)	0.084	8.14	90

*estimation is significant at both the 5% and 1% level of significance

Table 9 - Returns to contrarian trading rules classified by *H*

Panel A – Profits from the average of all trading rules (daily and weekly)

H grouping	Total (%)	BB (20,2) (%)	BB (20,1) (%)	BB (30,2) (%)	N
0.5 > *H*	11.319	10.515	10.995	12.445	20
0.5 < *H* > 0.55	9.718	9.700	8.673	10.781	26
0.55 < *H*	5.612	4.786	7.893	4.157	14

Panel B – Profits from the average of all trading rules (daily)

H grouping	Total (%)	BB (20,2) (%)	BB (20,1) (%)	BB (30,2) (%)	N
0.5 > *H*	5.426	5.310	6.550	4.420	10
0.5 < *H* > 0.55	1.034	-0.037	1.050	2.088	16
0.55 < *H*	2.498	3.325	4.675	-0.500	4

Panel C – Profits from the average of all trading rules (weekly)

H grouping	Total (%)	BB (20,2) (%)	BB (20,1) (%)	BB (30,2) (%)	N
0.5 > *H*	17.211	15.720	15.440	20.470	10
0.5 < *H* > 0.55	23.613	25.280	20.870	24.690	10
0.55 < *H*	6.183	5.370	9.180	6.020	10

Table 10 - Estimation results from regression of profits from contrarian trading rules on *H*.

This table presents the results of the regression between the profits generated by technical analysis on a time series and long-term dependencies (*H* as proxy), as expressed by the following equation:

$$\text{TTR Profits} = a_o + a_1\text{Hurst Exponent}(H) + \varepsilon_1$$

Panel A – Profits from the average of all trading rules

Estimation	Intercept	Hurst	Adjusted R^2	F-value	N
1 – *H*	77.56 (1.96)	131.79* (1.73)	0.05	3.01	60

Hurst is the nature of the dependencies in the time series as measured by the Hurst exponent
*estimation is significant at the 10% level of significance

Table 11 - Estimation results from regression of profits from technical analysis on lagged *H*

This table presents the results of the regression between the profits generated by technical analysis on time series (t_0) and the time series' long-term dependencies (*H* as proxy) in the past period (t-1), as expressed by the following equation:

$$\text{TTR Profits}_1 = a_o + a_2\text{Hurst Exponent } (H)_{(t-1)} + \varepsilon_1$$

Panel A – All observations

Estimation	Intercept	*H*	Adjusted R^2	F-value	N
1 – daily and weekly	0.52 (83.9)	-0.013 (-0.78)	-0.003	0.612	120
2 – daily	0.522 (83.49)	0.047 (0.876)	-0.003	0.767	60
3 – weekly	0.536 (48.12)	-0.011 (-0.533)	-0.012	0.284	60

*estimation is significant at both the 5% and 1% level of significance

Panel B – Observations with 10 lowest variance of *H* across sub-periods

Estimation	Intercept	*H*	Adjusted R^2	F-value	N
1 – daily and weekly	-1.44 (1.46)	1.86 (1.30)	0.017	1.70	40
2 – daily	-1.14 (-2.51)	2.11* (2.47)	0.213	6.14	20
3 – weekly	-1.88 (-1.45)	2.955 (1.28)	0.033	1.86	20

*estimation is significant at both the 5% and 1% level of significance

Table 12 - Estimation results from regression of sub-period *H* σ and σ full time series.

This table presents the results of the regression between the standard deviation of the *H* across sub-periods and the standard deviation of the log returns of the entire time series, as expressed by the following equation:

$$H\sigma = a_o + a_1\text{Data Set}\sigma + \varepsilon_1$$

Estimation	Intercept	Data set σ	Adjusted R^2	F-value	ρ (Pearson)	N
1	0.818 (21.36)	-37.26 (-14.42)	0.778	208.03	-0.88	60

*estimation is significant at both the 5% and 1% level of significance

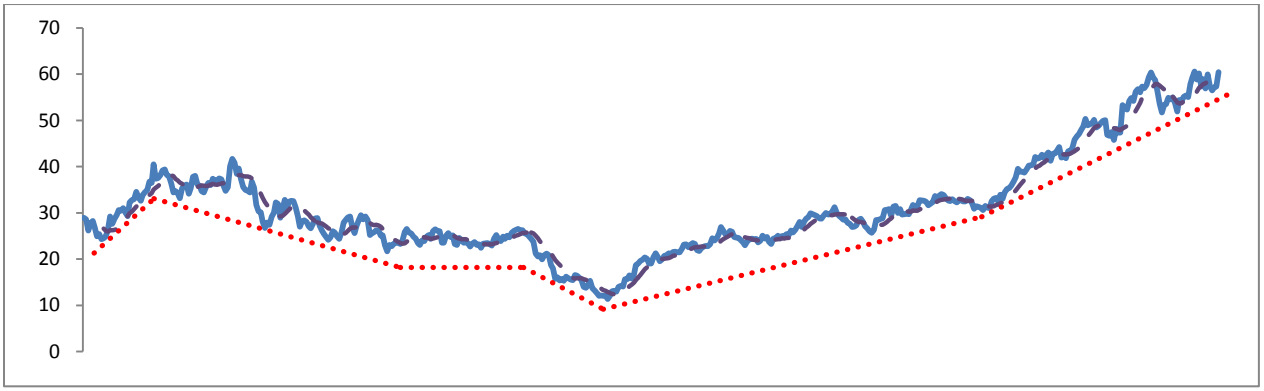
Table 13 – Summary of profits on daily versus weekly scale

	Profitable rules		Average return		Range (Max/Min)		σ of profits	
	Daily	Weekly	Daily (%)	Weekly (%)	Daily (%)	Weekly (%)	Daily (%)	Weekly (%)
MAC-O	33 (/ 90)	57 (/ 90)	-2.2	-3.1	21.3 / (21.9)	67.7 / (74.4)	9.4	35.2
Filter	15 (/ 90)	29 (/ 90)	-10.6	-22.6	21.9 / (33.9)	89.1 / (146.5)	10.7	39.1
BB	67 (/ 90)	63 (/ 90)	2.7	15.9	13.3 / 9.73	111.6 / -45.47	5.07	33.9
TRB-O	46 (/ 90)	52 (/ 90)	-0.2	10.8	17.4 / (17.3)	90.4 / (59.9)	8.1	29.1
Total	161 (/ 360)	201 (/ 360)	-2.58	0.25				

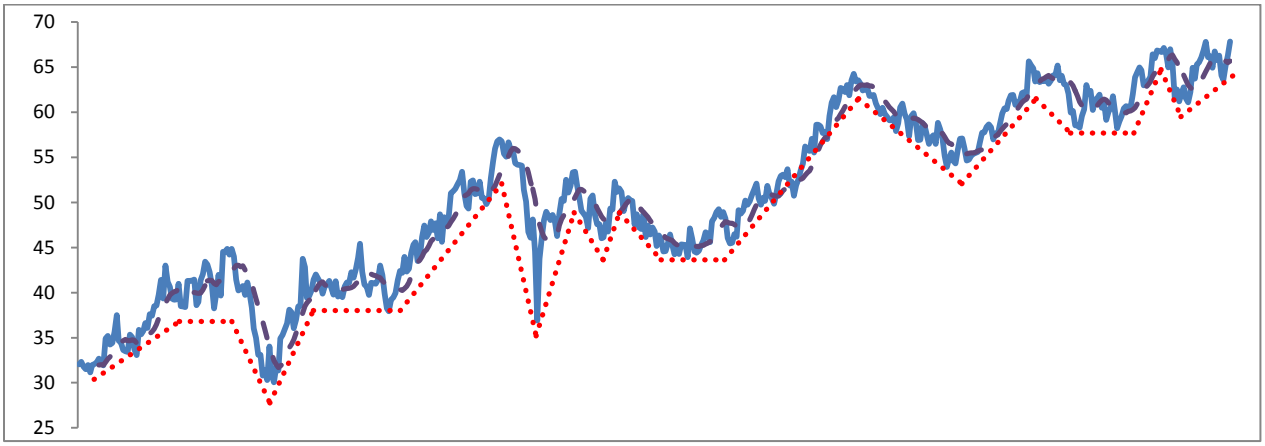
Figures

Figure 1 – Line graph of persistent, anti-persistent, and no persistence time series

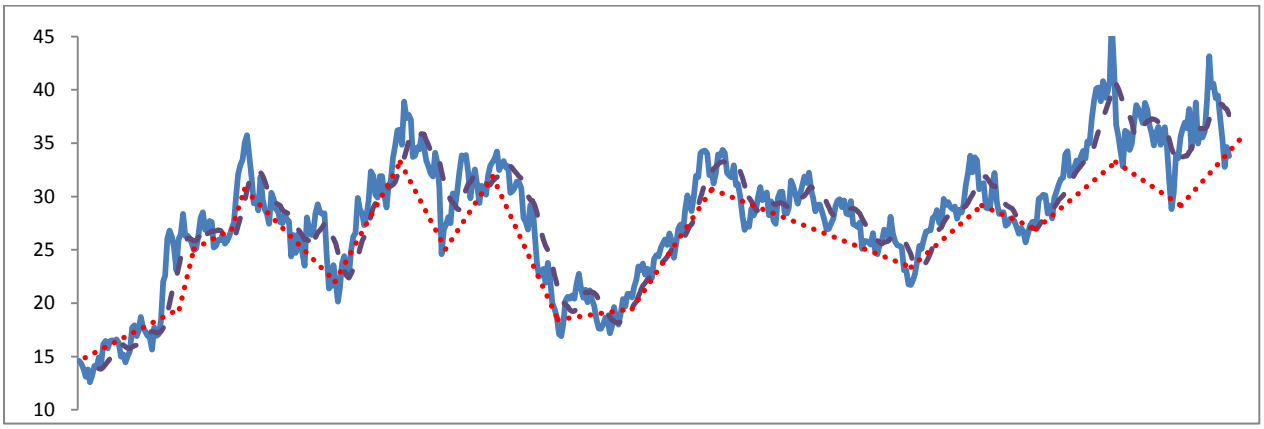
Panel A – Persistent time series (McDonalds, $H = 0.629$)



Panel B – Anti-persistent time series (Johnson and Johnson, $H = 0.464$)



Panel C – Time series with no persistence (Alcoa, $H = 0.498$)



Solid line – raw prices
Dashed line – 10-day moving average
Dotted line – support trend line

Figure 2 - Rescaled range analysis to estimate Hurst Exponent for DJIA time series